SESAM EXAMPLE

Linearized Buckling, P-Delta and Stress Stiffening Analyses of Frame Structure

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1 Introduction

This document is about the three analysis types: linearized buckling, P-Δ (P-delta) and stress stiffening. The example is named Jacket_LinBck_Pdelta_StrStiff and is run as a job in Sesam Manager. See **Figure 1-2** below.

The model shown in **Figure 1-1** below is an ocean frame structure. Four mass points and four beams connecting them represent a structure on top of the legs. Spring-to-ground elements fix the model at the bottom of the legs.

The example is based on units kN and m. Acceleration of gravity is set to 9.80665. The following programs and versions are used: GeniE 8.5, Wajac 7.9, Sestra 10.17, and Xtract 6.0.

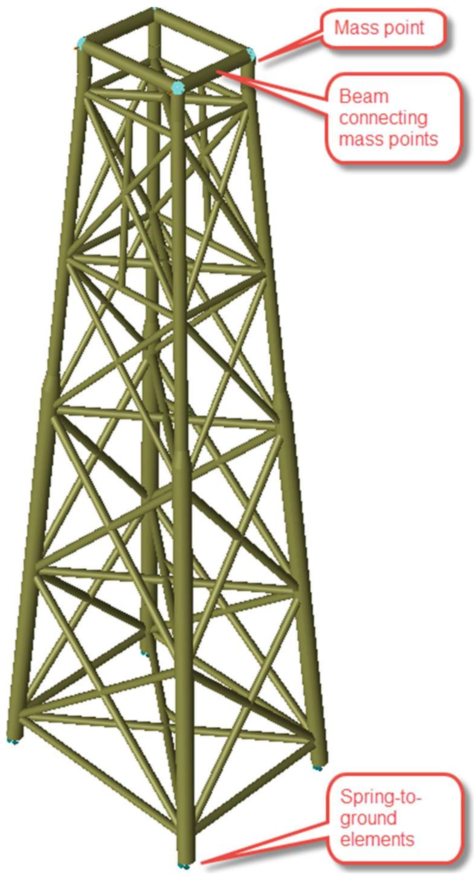

Figure 1-1 Frame structure for linearized buckling, P-Δ and stress stiffening analyses

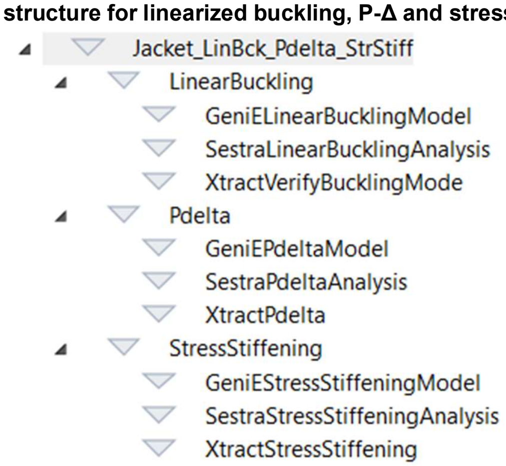

Figure 1-2 Sesam Manager workflow shown as 'Tree View'

2 Linearized Buckling Analysis

Sestra offers linearized buckling analysis as described in the Sestra user manual. Linearized buckling analysis is triggered by setting the LBUCK parameter as 1 on the CMAS command. The EIGA command demanding computation of eigenvalues (of which only the first is normally of interest) and with a shift value is also required.

A linearized buckling analysis is about determining the factor to be applied to a given load distribution so that buckling will occur. The factor is termed 'stability factor' or 'critical load level'. The functionality in Sestra is available for beam and plate/shell structures.

Linearized buckling analysis is a two-step procedure automatically performed by Sestra:

1. Static analysis is performed for the load for which the stability factor is desired.
    - **K** is the ordinary linear stiffness matrix.
    - Geometric stiffness matrix $\mathbf{K}_g$ is found based on stresses in plates/shells and forces in beams.
2. Buckling modes **Φ** and corresponding eigenvalues λ are found by solving the eigenvalue problem: $(\mathbf{K} - \lambda\mathbf{K}_g)\mathbf{\Phi} = \mathbf{0}$
    - The eigenvalues λ are the sought stability factors.
    - Only the first eigenvalue (buckling mode) and the corresponding stability factor are of interest.

A linearized buckling analysis may be used to find the buckling load P_E used in a P-Δ analysis. The stability factor corresponds to the ratio P_E/P in the formula for amplification factor α, see section 3.

Setting up a linearized buckling analysis is done as follows:

- Ensure that the load causing compression is the last load case or load combination. Alternatively, the command GSTF can be used to select the buckling load.
- Set parameter LBUCK on command CMAS equal to 1.
- Add command EIGA with an appropriate shift value.

Example of Sestra input for a linearized buckling analysis:

```
COMM   CHCK  ANTP  MSUM  MOLO  STIF  RTOP  LBCK
CMAS     0.    1.    1.    0.    0.    0.    1.
ITOP     1.
COMM    ENR                                                         SHIFT
EIGA     2.                                                         0.1
Z
```

In the present analysis a stability factor of 6.754 is found (in the sestra.lis file) for a combination of gravity and buoyancy. I.e. the frame structure will collapse when gravity minus buoyancy is multiplied by 6.754.

3 P-Δ Analysis

The P-Δ effect is a 2nd order effect: A horizontal force H gives moment M_H and horizontal displacement Δ. The vertical force P moves with the horizontal displacement. This causes an additional moment PΔ and an additional horizontal displacement.

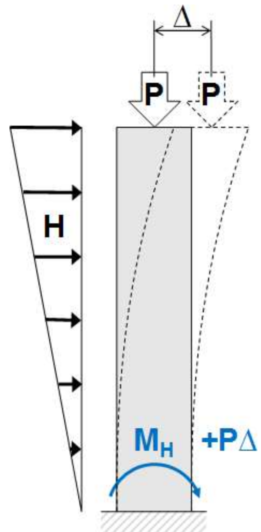

Figure 3-1 Horizontal force gives horizontal displacement that combined with vertical force produces additional moment

A proposal for a P-Δ analysis is as follows:

- For a vertical structure with uniform cross-sectional properties over the height – in essence a beam – compute the buckling load P_E:
  - $P_E = \pi^2 EI/(kl)^2$
  - The section modulus I may be found by hand calculation.
  - The buckling factor k depends on the boundary conditions of the beam as shown in **Figure 3-2** below. The figure to the left with k=2 is the one corresponding to the present structure.

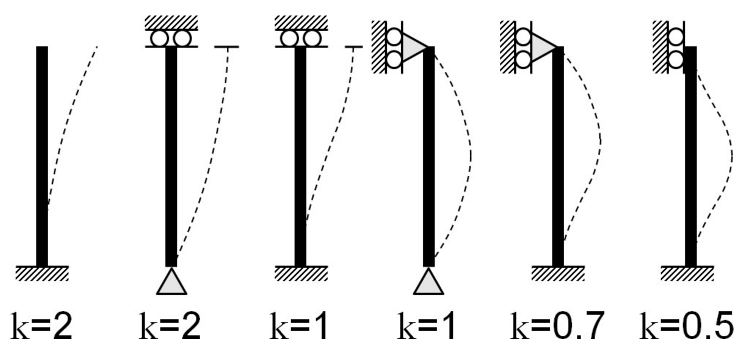

Figure 3-2 Buckling factors depending on boundary conditions

  - Find the horizontal displacement amplification factor $\alpha = 1/(1 – P/P_E)$.
- For structures that cannot be regarded as beams with uniform sectional properties over the height a linearized buckling analysis, see section 2, may be performed. In this case the stability factor corresponds to the ratio P_E/P.

- A static linear analysis is performed with wave, current, wind and other horizontal loads to find the horizontal displacement Δ at top.
- Apply a horizontal load (scale an arbitrary load) at top to produce an additional horizontal displacement (α–1)Δ – the total horizontal displacement is then αΔ.

In the present case a linearized buckling analysis has been performed, see section 2, and the stability factor has been found to be 6.754. This gives amplification factor α = 1/(1–1/6.754) = 1.174.

A single wave propagating in direction 270 degrees combined with a current is analyzed. Correspondingly, a horizontal load in –Y direction is defined (four point forces in the four top nodes). By viewing the results in Xtract the horizontal displacement in a selected node at top is found and used to compute the scaling factor. The node at top with negative X and Y coordinate values is chosen.

- The horizontal load gives a horizontal displacement (component Y) of –0.06344, see the upper left figure in **Figure 3-3** below.
- Wave plus current is combined in Xtract with gravity and buoyancy (Xtract load combination GravityWaveBuoyancy), this combination gives a horizontal displacement (component Y) Δ = –0.2409, see the upper right figure below.
- This means that the scaling factors for the load is: (α–1)Δ/(–0.06344) = (1.174–1)*(–0.2409)/(–0.06344) = 0.6607.
- A load combination (GravityWaveBuoyancyPlusPdelta) is made in Xtract by adding the horizontal load times 0.6607 to the wave + current + gravity + buoyancy load combination. This new combination is used in further calculations.
- The combination gives displacement (component Y) = –0.2828.

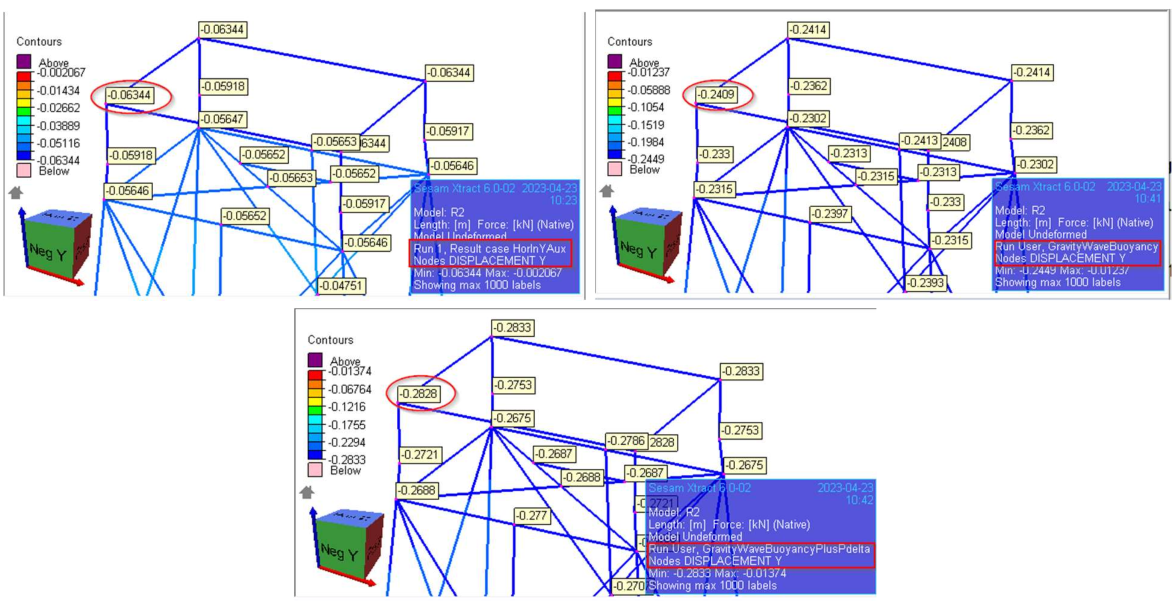

Figure 3-3 Y-displacements for horizontal forces and two load combinations

4 Stress Stiffening Analysis

Sestra offers stress stiffening analysis as described in the Sestra user manual. In such an analysis the stiffening effect of tensile stresses/forces and softening effect of compressive stresses/forces are accounted for. (A more descriptive name of the method would thus be 'stress stiffening/softening analysis'.) The functionality in Sestra is available for beam and plate/shell structures.

Stress stiffening analysis is a two-step procedure automatically performed by Sestra:

1. Static analysis of load (typically gravity) is performed to find initial stresses in plates/shells and forces in beams.
    - Geometric stiffness (or initial stress) matrices of the basic elements are calculated based on initial stresses.
2. Geometric stiffness matrices are added to the ordinary stiffness matrices of the basic elements.
    - An updated stiffness matrix for the model is achieved and based on this a new analysis is performed – this may be a static, free vibration or forced dynamic analysis.

A stress stiffening analysis may be highly relevant in an eigenvalue analysis since the softening of a slender structure due to compressive stresses and forces may significantly increase the eigenperiods (natural periods).

Setting up a stress stiffening analysis is done as follows:

- Ensure that the load causing tension/compression is the last load case or load combination. Alternatively, the command GSTF can be used to select the stress stiffening load.
- Set parameter STIF on command CMAS equal to 2.

Example of Sestra input for a static analysis with stress stiffening:

```
COMM  CHCK ANTP MSUM MOLO STIF
CMAS    0.   1.   1.   0.   2.
ITOP    3.
Z
```

Example of Sestra input for an eigenvalue analysis computing 10 modes with stress stiffening:

```
COMM CHCK ANTP MSUM MOLO STIF
CMAS   0.   2.   1.   0.   2.
ITOP   3.
EIGA  10.
IDTY   3.
DYMA   1.
Z
```

In the present analysis a static analysis of a wave plus current load combined with gravity and buoyancy shall be performed with the gravity plus buoyancy as stress stiffening case. Therefore, a combination of the gravity load case and the buoyancy load case is made. Smart load combinations must be switched off in GeniE (edit the meshing activity) to make the load combination available to Sestra. The load causing tension/compression will then be the last load case as required by a stress stiffening analysis (unless command GSTF is used).

The stress stiffening analysis gives displacement of the selected node at top with negative X and Y coordinate values to be –0.2571 (in Y), see **Figure 4-1** below. The P-delta analysis gave –0.2828, i.e. the P-delta analysis is somewhat more conservative than the stress stiffening analysis by a factor of –0.2828/(–0.2571) = 1.100.

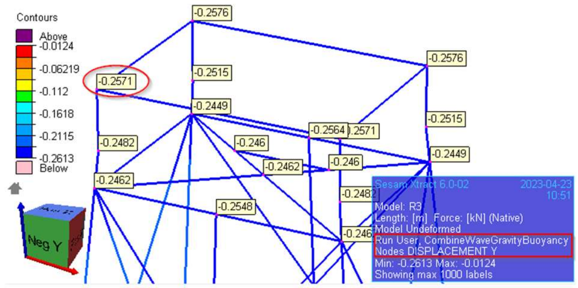

Figure 4-1 Y-displacements for wave+current, gravity and buoyancy combined

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